

## Algebraic equation manipulation for electric circuits

This worksheet and all related files are licensed under the Creative Commons Attribution License, version 1.0. To view a copy of this license, visit <http://creativecommons.org/licenses/by/1.0/>, or send a letter to Creative Commons, 559 Nathan Abbott Way, Stanford, California 94305, USA. The terms and conditions of this license allow for free copying, distribution, and/or modification of all licensed works by the general public.

---

Resources and methods for learning about these subjects (list a few here, in preparation for your research):

## Questions

---

### Question 1

Solve for the value of  $x$  in the following equations:

$$10x = 80 \quad x =$$

$$3 = \frac{15}{x} \quad x =$$

file 03322

---

### Question 2

Solve for the value of  $x$  in the following equations:

$$5x = 15 \quad x =$$

$$6 = \frac{x}{2} \quad x =$$

file 03323

---

### Question 3

Solve for the value of  $a$  in the following equations:

Equation 1:  $a - 4 = 10$

Equation 2:  $30 = a + 3$

Equation 3:  $-2a = 9$

Equation 4:  $\frac{a}{4} = 3.5$

file 03062

---

### Question 4

Solve for  $n$  in the following equations:

Equation 1:  $-56 = -14n$

Equation 2:  $54 - n = 10$

Equation 3:  $\frac{4}{n} = 12$

Equation 4:  $28 = 2 - n$

file 03063

---

Question 5

Solve for the value of  $x$  in the following equations:

$$\frac{x + 5}{2} = 20 \quad x =$$

$$6 = \sqrt{x - 2} \quad x =$$

[file 03324](#)

---

Question 6

Solve for the value of  $x$  in the following equations:

$$2(x + 5) = 36 \quad x =$$

$$3 = \sqrt{2 - x} \quad x =$$

[file 03325](#)

---

Question 7

Manipulate each of these equations to solve for  $a$ :

$$\frac{b - a}{c} = d \quad \sqrt{a + b} = c^2 d$$

[file 03328](#)

---

Question 8

Manipulate each of these equations to solve for  $a$ :

$$\frac{a - b}{c} = d^2 \quad b + a^2 = \frac{c}{d}$$

[file 03329](#)

---

Question 9

The formula for calculating total resistance of three series-connected resistors is as follows:

$$R = R_1 + R_2 + R_3$$

Algebraically manipulate this equation to solve for one of the series resistances ( $R_1$ ) in terms of the other two series resistances ( $R_2$  and  $R_3$ ) and the total resistance ( $R$ ). In other words, write a formula that solves for  $R_1$  in terms of all the other variables.

[file 03066](#)

---

Question 10

Manipulate this equation to solve for resistor value  $R_1$ , given the values of  $R_2$  and  $R_{parallel}$ :

$$R_{parallel} = \frac{R_1 R_2}{R_1 + R_2}$$

Then, give an example of a practical situation where you might use this new equation.

[file 01988](#)

---

---

Question 11

The formula for calculating total resistance of three parallel-connected resistors is as follows:

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Algebraically manipulate this equation to solve for one of the parallel resistances ( $R_1$ ) in terms of the other two parallel resistances ( $R_2$  and  $R_3$ ) and the total resistance ( $R$ ). In other words, write a formula that solves for  $R_1$  in terms of all the other variables.

[file 03067](#)

---

Question 12

The following equations solve for the output voltage of various switching converter circuits (unloaded), given the switch duty cycle  $D$  and the input voltage:

$$V_{out} = D V_{in} \quad (\text{Buck converter circuit})$$

$$V_{out} = \frac{V_{in}}{1 - D} \quad (\text{Boost converter circuit})$$

$$V_{out} = \frac{D V_{in}}{1 - D} \quad (\text{Inverting or Cuk converter circuit})$$

Manipulate each of these equations to solve for duty cycle ( $D$ ) in terms of the input voltage ( $V_{in}$ ) and desired output voltage ( $V_{out}$ ). Remember that duty cycle is always a quantity between 0 and 1, inclusive.

[file 02161](#)

---

Question 13

The power dissipation of a transistor is given by the following equation:

$$P = I_C \left( V_{CE} + \frac{V_{BE}}{\beta} \right)$$

Manipulate this equation to solve for beta, given all the other variables.

[file 00502](#)

---

Question 14

The equation for voltage gain ( $A_V$ ) in a typical inverting, single-ended opamp circuit is as follows:

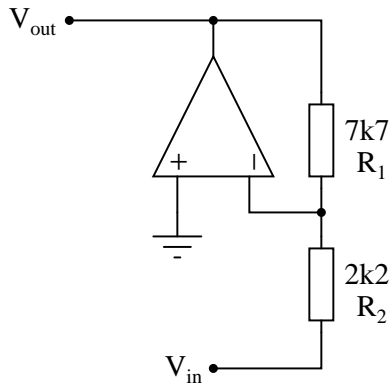
$$A_V = \frac{R_1}{R_2}$$

Where,

$R_1$  is the feedback resistor (connecting the output to the inverting input)

$R_2$  is the other resistor (connecting the inverting input to voltage signal input terminal)

Suppose we wished to change the voltage gain in the following circuit from 3.5 to 4.9, but only had the freedom to alter the resistance of  $R_2$ :



Algebraically manipulate the gain equation to solve for  $R_2$ , then determine the necessary value of  $R_2$  in this circuit to give it a voltage gain of 4.9.

file 02708

---

**Question 15**

The equation for voltage gain ( $A_V$ ) in a typical noninverting, single-ended opamp circuit is as follows:

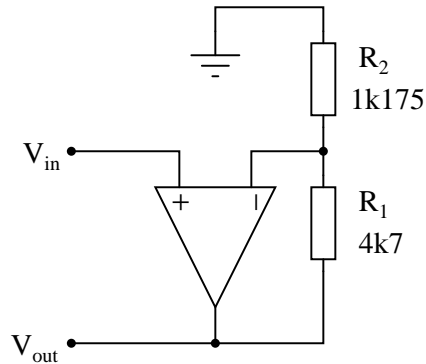
$$A_V = \frac{R_1}{R_2} + 1$$

Where,

$R_1$  is the feedback resistor (connecting the output to the inverting input)

$R_2$  is the other resistor (connecting the inverting input to ground)

Suppose we wished to change the voltage gain in the following circuit from 5 to 6.8, but only had the freedom to alter the resistance of  $R_2$ :



Algebraically manipulate the gain equation to solve for  $R_2$ , then determine the necessary value of  $R_2$  in this circuit to give it a voltage gain of 6.8.

[file 02707](#)

---

**Question 16**

The decay of a variable over time in an RC or LR circuit follows this mathematical expression:

$$e^{-\frac{t}{\tau}}$$

Where,

$e$  = Euler's constant ( $\approx 2.718281828$ )

$t$  = Time, in seconds

$\tau$  = Time constant of circuit, in seconds

For example, if we were to evaluate this expression and arrive at a value of 0.398, we would know the variable in question has decayed from 100% to 39.8% over the period of time specified.

However, calculating the amount of time it takes for a decaying variable to reach a specified percentage is more difficult. We would have to manipulate the equation to solve for  $t$ , which is part of an exponent.

Show how the following equation could be algebraically manipulated to solve for  $t$ , where  $x$  is the number between 0 and 1 (inclusive) representing the percentage of original value for the variable in question:

$$x = e^{-\frac{t}{\tau}}$$

Note: the "trick" here is how to isolate the exponent  $-\frac{t}{\tau}$ . You will have to use the natural logarithm function!

[file 02001](#)

---

Question 17

The electrical resistance of a conductor at any temperature may be calculated by the following equation:

$$R_T = R_r + R_r\alpha T - R_r\alpha T_r$$

Where,

$R_T$  = Resistance of conductor at temperature  $T$

$R_r$  = Resistance of conductor at reference temperature  $T_r$

$\alpha$  = Temperature coefficient of resistance at reference temperature  $T_r$

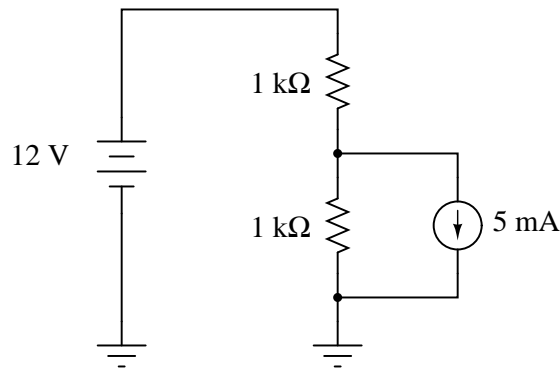
Simplify this equation by means of factoring.

[file 00509](#)

---

Question 18

Calculate all currents in this DC circuit:



Hint: it may help you to set up the necessary equation by labeling the current through the lower resistor as  $I$  and the current through the upper resistor as  $I + 0.005$ .

[file 03116](#)

---

Question 19

Voltage and current gains, expressed in units of decibels, may be calculated as such:

$$A_{V(dB)} = 10 \log (A_{V(ratio)})^2$$

$$A_{I(dB)} = 10 \log (A_{I(ratio)})^2$$

Another way of writing this equation is like this:

$$A_{V(dB)} = 20 \log A_{V(ratio)}$$

$$A_{I(dB)} = 20 \log A_{I(ratio)}$$

What law of algebra allows us to simplify a logarithmic equation in this manner?

[file 00830](#)

## Answers

---

Answer 1

$$10x = 80 \quad x = 8$$

$$3 = \frac{15}{x} \quad x = 5$$

---

Answer 2

$$5x = 15 \quad x = 3$$

$$6 = \frac{x}{2} \quad x = 12$$

---

Answer 3

Equation 1:  $a = 14$

Equation 2:  $a = 27$

Equation 3:  $a = -4.5$

Equation 4:  $a = 14$

---

Answer 4

Equation 1:  $n = 4$

Equation 2:  $n = 44$

Equation 3:  $n = 0.\overline{333}$

Equation 4:  $n = -26$

---

Answer 5

$$\frac{x+5}{2} = 20 \quad x = 35$$

$$6 = \sqrt{x-2} \quad x = 38$$

---

Answer 6

$$2(x+5) = 36 \quad x = 13$$

$$3 = \sqrt{2-x} \quad x = -7$$

---

Answer 7

$$a = b - cd$$

$$a = c^4 d^2 - b$$



---

Answer 8

$$a = cd^2 + b$$

$$a = \sqrt{\frac{c}{d} - b}$$

---

Answer 9

$$R_1 = R - (R_2 + R_3) \quad \text{or} \quad R_1 = R - R_2 - R_3$$

---

Answer 10

$$R_1 = \frac{R_2 R_{parallel}}{R_2 - R_{parallel}}$$

I'll let you figure out a situation where this equation would be useful!

---

Answer 11

$$R_1 = \frac{1}{\frac{1}{R} - \left(\frac{1}{R_2} + \frac{1}{R_3}\right)} \quad \text{or} \quad R_1 = \frac{1}{\frac{1}{R} - \frac{1}{R_2} - \frac{1}{R_3}}$$

---

Answer 12

$$D = \frac{V_{out}}{V_{in}} \quad (\text{Buck converter circuit})$$

$$D = 1 - \left(\frac{V_{in}}{V_{out}}\right) \quad (\text{Boost converter circuit})$$

$$D = \frac{V_{out}}{V_{in} + V_{out}} \quad (\text{Inverting or Cuk converter circuit})$$

---

Answer 13

$$\beta = \frac{V_{BE}}{\frac{P}{I_C} - V_{CE}}$$

---

Answer 14

$$R_2 = \frac{R_1}{A_V}$$

For the circuit shown,  $R_2$  would have to be set equal to 1.571 k $\Omega$ .

---

Answer 15

$$R_2 = \frac{R_1}{A_V - 1}$$

For the circuit shown,  $R_2$  would have to be set equal to 810.3  $\Omega$ .

---

Answer 16

Showing all the necessary steps:

$$x = e^{-\frac{t}{\tau}}$$

$$\ln x = \ln \left( e^{-\frac{t}{\tau}} \right)$$

$$\ln x = -\frac{t}{\tau}$$

$$t = -\tau \ln x$$

---

Answer 17

$$R_T = R_r[1 + \alpha(T - T_r)]$$

Follow-up question: when plotted on a graph with temperature ( $T$ ) as the independent variable and resistance ( $R_T$ ) as the dependent variable (i.e. a two-axis graph with  $T$  on the horizontal and  $R$  on the vertical), is the resulting plot linear? Why or why not? How is it possible to tell just by looking at the equation, prior to actually plotting on a graph?

---

Answer 18

Upper resistor current = 8.5 mA; lower resistor current = 3.5 mA

---

Answer 19

$$\log a^b = b \log a$$

Challenge question: knowing this algebraic law, solve for  $x$  in the following equation:

$$520 = 8^x$$

## Notes

---

### Notes 1

Have your students come to the front of the class and show everyone else the techniques they used to solve for the value of  $x$  in each equation. Remind them to document each and every step in the process, so that nothing is left to guess or to chance.

---

### Notes 2

Have your students come to the front of the class and show everyone else the techniques they used to solve for the value of  $x$  in each equation. Remind them to document each and every step in the process, so that nothing is left to guess or to chance.

---

### Notes 3

Have your students come to the front of the class and show everyone else the techniques they used to solve for the value of  $a$  in each equation. Remind them to document each and every step in the process, so that nothing is left to guess or to chance.

---

### Notes 4

Have your students come to the front of the class and show everyone else the techniques they used to solve for the value of  $a$  in each equation. Remind them to document each and every step in the process, so that nothing is left to guess or to chance.

Equations 2 through 4 require two steps to solve for  $n$ . Equation 1 only requires a single step, but the two negative numbers may be a bit confusing to some.

---

### Notes 5

Have your students come to the front of the class and show everyone else the techniques they used to solve for the value of  $x$  in each equation. Remind them to document each and every step in the process, so that nothing is left to guess or to chance.

---

### Notes 6

Have your students come to the front of the class and show everyone else the techniques they used to solve for the value of  $x$  in each equation. Remind them to document each and every step in the process, so that nothing is left to guess or to chance.

---

### Notes 7

Have your students come to the front of the class and show everyone else the techniques they used to solve for  $a$  in each equation. Remind them to document each and every step in the process, so that nothing is left to guess or to chance.

---

### Notes 8

Have your students come to the front of the class and show everyone else the techniques they used to solve for  $a$  in each equation. Remind them to document each and every step in the process, so that nothing is left to guess or to chance.

---

### Notes 9

This question is nothing more than practice algebraically manipulating equations. Ask your students to show you how they solved it, and how the two given answers are equivalent.

---

### Notes 10

This question is really nothing more than an exercise in algebraic manipulation.

---

Notes 11

This question is nothing more than practice algebraically manipulating equations. Ask your students to show you how they solved it, and how the two given answers are equivalent.

---

Notes 12

Given the equations for these converter circuit types solving for output voltage in terms of input voltage and duty cycle  $D$ , this question is nothing more than an exercise in algebraic manipulation.

Note to your students that all of these equations assume a condition of zero load on the converter circuit. When loads are present, of course, the output voltage will not be the same as what is predicted by these neat, simple formulae. Although these DC-DC power converter circuits are commonly referred to as "regulators," it is somewhat misleading to do so because it falsely implies a capacity for self-correction of output voltage. Only when coupled to a feedback control network are any of these converter circuits capable of actually *regulating* output voltage to a set value.

---

Notes 13

Although this question is essentially nothing more than an exercise in algebraic manipulation, it is also a good lead-in to a discussion on the importance of power dissipation as a semiconductor device rating.

High temperature is the bane of most semiconductors, and high temperature is caused by excessive power dissipation. A classic example of this, though a bit dated, is the temperature sensitivity of the original germanium transistors. These devices were extremely sensitive to heat, and would fail rather quickly if allowed to overheat. Solid state design engineers had to be very careful in the techniques they used for transistor circuits to ensure their sensitive germanium transistors would not suffer from "thermal runaway" and destroy themselves.

Silicon is much more forgiving than germanium, but heat is still a problem with these devices. At the time of this writing (2004), there is promising developmental work on silicon carbide and gallium nitride transistor technology, which is able to function under *much* higher temperatures than silicon.

---

Notes 14

Nothing more than a little algebra to obtain the answers for this question!

---

Notes 15

Nothing more than a little algebra to obtain the answers for this question!

In my experience, most American high school graduates are extremely weak in logarithms. Apparently this is not taught very well at the high school level, which is a shame because logarithms are a powerful mathematical tool. You may find it necessary to explain to your students what a logarithm is, and exactly why it "un-does" the exponent.

When forced to give a quick presentation on logarithms, I usually start with a generic definition:

$$\text{Given: } b^a = c$$

$$\text{Logarithm defined: } \log_b c = a$$

Verbally defined, the logarithm function asks us to find the power ( $a$ ) of the base ( $b$ ) that will yield  $c$ .

Next, I introduce the common logarithm. This, of course, is a logarithm with a base of 10. A few quick calculator exercises help students grasp what the common logarithm function is all about:

$$\log 10 =$$

$$\log 100 =$$

$$\log 1000 =$$

$$\log 10000 =$$

$$\log 100000 =$$

$$\log \frac{1}{10} =$$

$$\log \frac{1}{100} =$$

$$\log \frac{1}{1000} =$$

After this, I introduce the *natural logarithm*: a logarithm with a base of  $e$  (Euler's constant):

$$\text{Natural logarithm defined: } \ln x = \log_e x$$

Have your students do this simple calculation on their calculators, and explain the result:

$$\ln 2.71828 =$$

Next comes an exercise to help them understand how logarithms can "undo" exponentiation. Have your students calculate the following values:

$$e^2 =$$

$$e^3 =$$

$$e^4 =$$

Now, have them take the natural logarithms of each of those answers. They will find that they arrive at the original exponent values (2, 3, and 4, respectively). Write this relationship on the board as such for your students to view:

$$\ln e^2 = 2$$

$$\ln e^3 = 3$$

$$\ln e^4 = 4$$

Ask your students to express this relationship in general form, using the variable  $x$  for the power instead of an actual number:

$$\ln e^x = x$$

It should now be apparent that the natural logarithm function has the ability to "undo" a power of  $e$ . Now it should be clear to your students why the given sequence of algebraic manipulations in the answer for this question is true.

---

Notes 17

Just an exercise in algebra here!

---

Notes 18

This is an interesting circuit to mathematically analyze. While not requiring simultaneous equations, or even a network theorem such as Superposition or Thévenin's, it is nevertheless a good exercise in applying the basic laws of circuit analysis (Ohm's, Kirchhoff's) and algebra.

---

Notes 19

Logarithms are a confusing, but powerful, algebraic tool. In this example, we see how the logarithm of a power function is converted into a simple multiplication function.

The challenge question asks students to apply this relationship to an equation not containing logarithms at all. However, the fundamental rule of algebra is that you may perform any operation (including logarithms) to any equation so long as you apply it equally to *both sides* of the equation. Logarithms allow us to take an algebra problem such as this and simplify it significantly.